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Determination of crack growth parameters of alumina in 4-point bending tests

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September 1985

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SUMMARY

Alumina containers are being considered for the disposal of nuclear waste in a number of countries. For this ceramic material subcritical crack growth is an potentially important failure mechanism.

The crack growth behaviour of one particular type of hot isostatically pressed alumina has been investigated at 70°C in a strongly corrosive brine using fracture-mechanical methods. The use of a recently developed method for interpreting lifetime measurements in static bending tests allowed a determination of crack growth rates as low as $10^{-11}$ m/s.

It was observed that the dependence of the crack growth rate on the stress intensity factor can be described in terms of a power law over the whole range investigated. The exponent in the power law was found to be close to $n = 20$. Published values for the same material, determined by the double torsion method, lie above $n = 100$. The discrepancy between these values is probably due to R-curve effects (i.e. to an increase of the critical stress intensity factor in the course of crack extension) that influence the behaviour of macroscopic cracks and that of natural cracks about 100 $\mu$m in size differently.

Because of the high lifetime required of containers for final disposal of radioactive waste the low values determined for the crack growth exponent imply that tensile stresses must be kept quite small.
ZUSAMMENFASSUNG

Für die nukleare Entsorgung werden in einigen Ländern Endlagerbehälter aus Aluminiumoxid in Erwägung gezogen. Bei diesem keramischen Material ist unterkritische Rissausbreitung ein wichtiger potentieller Versagensmechanismus.

Es wurde ein heissisostatisch gepresstes Al$_2$O$_3$ hinsichtlich seines Risswachstumsverhaltens in einer stark korrosiven Salzlösung bei 70°C mit bruchmechanischen Methoden untersucht. Durch eine neuartige Auswertung von Lebendauermessungen im statischen Biegeversuch konnten Risswachstums- geschwindigkeiten bis zu $10^{-11}$ m/s gemessen werden.


Bei den hohen anzustrebenden Lebensdauern von Endlagerbehältern führt der niedrige gemessene Wert des Risswachstumsexponenten zu recht tiefen Werten für die zulässigen Zugspannungen.
Dans plusieurs pays on étudie la possibilité d'utiliser des conteneurs en alumine pour l'entreposage final des déchets radioactifs. Dans ce type de matériaux céramiques la croissance sous-critique des fissures représente un mécanisme important pouvant conduire à la rupture du conteneur.

On a étudié, par les méthodes de la mécanique de la rupture, la propagation des fissures, à 70°C et dans une solution fortement corrosive, dans un type particulier d'alumine pressé isostatiquement à chaud. A l'aide d'une méthode récemment développée d'évaluation des essais de durée de vie sous flexion, on a pu mesurer des taux de propagation de fissure allant jusqu'à $10^{-11}$ m/s.

On a constaté que la vitesse de propagation des failles est proportionnelle au facteur d'intensité à la puissance $n \approx 20$, dans l'ensemble du domaine considéré. Dans la littérature on trouve des valeurs $n > 100$ obtenues antérieurement pour le même matériau par la méthode de double torsion. Il est probable que la disparité entre ces deux résultats est due à des effets de courbe R (c.-à-d. à une augmentation du facteur critique d'intensité au cours de la propagation de la fissure): cet effet aurait une influence sur le comportement des macrofissures différente de celle qu'il a sur la propagation des fissures naturelles, de l'ordre de grandeur de 100 μm.

Si l'on tient compte de la durée de vie requise pour un conteneur pour l'entreposage final de déchets radioactifs, on constate que la faible valeur mesurée pour l'exposant de croissance impose de limiter fortement les contraintes en tension.
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1. **INTRODUCTION**

For ultimate storage of high level waste, the use of aluminium oxide as container material is being considered in several countries. This type of containers might fail due to corrosion, buckling and by exceeding the tensile strength or subcritical crack growth because tensile stresses cannot be excluded. Since water will affect the container surface and lifetimes of the order of 1000 years are required, it seems that subcritical crack growth of preexisting surface cracks is the most serious potential failure mechanism. This study deals with the subcritical crack growth behaviour of a hot isostatically pressed alumina envisaged for ultimate storage purposes /1/. As fracture in ceramics usually starts at a defect introduced in the course of fabrication or processing, it is necessary to study the crack growth phenomenon starting from natural cracks.

2. **FUNDAMENTAL EQUATIONS**

In case of linear-elastic material behaviour crack growth is governed by the stress intensity factor $K_I$, defined by

$$K_I = \sigma \sqrt{a} Y$$

(1)

where $a$ is the size of a crack and $Y$ is a geometric correction factor dependent on the shape of the crack and of the specimen. If $v(K_I)$ is the crack growth rate

$$\frac{da}{dt} = v(K_I)$$

(2)

the lifetime of a specimen with a surface crack, by combination of Eqs. (1) and (2), becomes

$$t_t = \frac{2}{Y^2 \sigma^2} \int_{K_{I,i}}^{K_{I,c}} \frac{K_I dK_I}{v(K_I)}$$

(3)

where

$$K_{I,i} = \sigma' \sqrt{a_0} Y$$

(4)

is the initial value of the stress intensity factor. It is assumed that $Y$ is independent of crack length between $a_0$ and the critical crack length $a_c$. By definition $K_{I,c}$ is the fracture toughness. It is often observed that the dependency of $v(K_I)$ on $K_I$ takes the form of a power law over a wide range of growth rates:
\[ v(K_1) = A K_1^n \]  \hspace{1cm} (5)

In this case one obtains for \[ K_{II}^{n-2} \ll K_{IC}^{n-2} \]

\[ t_f = B \sigma_c^{n-2} \sigma^{-n} \]  \hspace{1cm} (6)

with

\[ B = \frac{2}{AY^2(n-2)K_{IC}^{n-2}} \]  \hspace{1cm} (7)

and the strength \( \sigma_c \) in the absence of subcritical crack growth

\[ \sigma_c = \frac{K_{IC}}{Y \sqrt{\sigma_0}} \]  \hspace{1cm} (8)

The distribution of ceramic strength values can often be described by a Weibull distribution. For the bending strength in an inert medium, \( \sigma_c \), the cumulative frequency \( F \), i.e. the probability that the actual strength of a randomly chosen sample lies below \( \sigma_c \), is given according to the equation

\[ F = 1 - \exp\left[-\left(\frac{\sigma_c}{\sigma'_0}\right)^m\right] \]  \hspace{1cm} (9)

where \( m \) and \( \sigma'_0 \) are the Weibull parameters. In a logarithmic plot according to

\[ \ln \ln \frac{1}{1-F} = m \ln \left(\frac{\sigma_c}{\sigma'_0}\right) = m \ln \sigma_c - m \ln \sigma'_0 \]  \hspace{1cm} (10)

a straight line with the slope \( m \) results.

Substituting \( t_f \) for \( \sigma_c \) using Eq. (6) results in a Weibull-distribution for the time to failure with the cumulative frequency

\[ F = 1 - \exp\left[-\left(\frac{t_f}{t'_0}\right)^{m^*}\right] \]  \hspace{1cm} (11)

with

\[ m^* = \frac{m}{n-2} \]  \hspace{1cm} (12a)

and

\[ t'_0 = B \sigma'_0^{n-2} \sigma^{-n} \]  \hspace{1cm} (12b)
3. METHODS OF DETERMINATION OF SUBCRITICAL CRACK GROWTH

In recent years various methods have been developed to determine the \( v-K_1 \)-behaviour.

a) Double-Torsion-method (DT)

The advantages of this most popular method are:
- Crack extension can be observed under the microscope or in an indirect manner by compliance measurements.
- A complete \( v-K_1 \)-curve can be determined with, in principle, only one specimen.

Disadvantages with respect to lifetime predictions are:
- Crack growth rates are limited by \( v > 10^{-9} \text{ m/s} \).
- DT-measurements are carried out with cracks of the order of several mm, but for lifetime predictions the crack growth behaviour of natural cracks of the order of 50 \( \mu \text{m} \) is of interest. The extrapolation of the results to natural crack sizes is not always possible.
- Especially for materials with a strong R-curve effect (i.e. materials exhibiting an increase of \( K_{IC} \) in the course of the crack extension) DT-measurements are not appropriate to predict the behaviour of small natural cracks where R-curve effects are negligible.

b) Dynamic bending tests

From measurements of bending strengths at different stress rates one can evaluate \( n \) and \( B \) (or \( A \)).

Advantages of this procedure are:
- The test method is very simple and only a simple equipment is necessary.
- Crack growth data are determined with specimens containing natural cracks and therefore problems of transferability are eliminated.

The disadvantages are:
- The type of \( v-K_1 \) relation has to be known.
- Inevitably, the bending strength is affected mainly by crack growth at a relatively high crack growth rate so that the crack growth parameters obtained are not necessarily characteristic of those crack growth rates which are of interest for lifetime predictions.

c) Crack growth data evaluated from lifetime measurements in static bending tests

From lifetime measurements performed at different bending stresses \( n \) and \( B \) can be determined by plotting \( \lg(t_f) \) versus \( \lg(d) \) using Eq. (6) or by evaluating the Weibull parameter \( m \) using Eq. (12a).

The advantages of this method - apart from those mentioned under b) - are:
- Lifetime predictions require only extrapolation of the measured lifetimes.
The crack growth rates which appear in lifetime measurements can be distinctly lower than those resulting from DT-measurements.

Disadvantage:
- For integration a particular type of \( v-K_{I} \) dependency has to be assumed and its accuracy can be judged in a supplementary step only.

d) Determination of \( v-K_{I} \)-relations from lifetime measurements

A modified procedure for the evaluation of crack growth rates from lifetime measurements is proposed in /6/. This method does not require the \( v-K_{I} \) dependency to be known.

The principle is to load a series of samples with a constant stress \( \sigma \).

By differentiation of Eq. (3) with respect to the initial stress intensity factor \( K_{II} \) one obtains

\[
dK_{II} = \frac{2K_{II}^{2}}{Y^{2}\sigma^{2}t_{f}v(K_{II})}
\]  

Inserting logarithmic derivatives yields

\[
d(ln t_{f}) = \frac{2K_{II}^{2}}{Y^{2}\sigma^{2}t_{f}v(K_{II})} = dK_{II}
\]  

and, consequently, the crack growth rate attached to \( K_{II} \) becomes

\[
v(K_{II}) = - \frac{2K_{II}^{2}}{Y^{2}\sigma^{2}t_{f}v(K_{II})} \frac{d(ln K_{II})}{d(ln t_{f})} 
\]  

In the special case of a power law one has

\[
\frac{d(ln t_{f})}{d(ln K_{II})} = n-2
\]

As

\[
K_{II}/K_{IC} = \sigma/\sigma_{c}
\]  

one obtains

\[
v(K_{II}/K_{IC}) = - \frac{2K_{IC}^{2}}{Y^{2}\sigma_{c}^{2}t_{f}} \frac{d(ln K_{II}/K_{IC})}{d(ln t_{f})} 
\]

The procedure is relatively simple. In a first series of tests \( N \) samples are loaded with a given stress \( \sigma \). The \( N \) values of lifetimes are ranked in increasing order. A second series also involving \( N \) specimens is tested in dynamic bending tests at high stress rates in an
inert environment to give the distribution of so-called "inert strength". These values are similarly ranked in increasing order. The \( \gamma \)-th value of lifetime \( t_f,\gamma \) is associated with the \( \gamma \)-th value of inert bending strength \( \sigma'_C,\gamma \). The latter is transformed to \( K_{II,\gamma}/K_{IC} \) using Eq. (16), with \( \sigma' \) from constant load tests.

By plotting \( t_f,\gamma \) versus \( K_{II,\gamma}/K_{IC} \) one obtains the dependency depicted schematically in Figure 1. Using a suitable smoothing procedure, one can determine \( d(ln \ t_f)/d(ln \ K_{II}/K_{IC}) \) for each value of \( K_{II}/K_{IC} \). From Eq. (17) the accompanying \( \nu-K_{I}-\)curve of Figure 2 results.

4. EXPERIMENTAL INVESTIGATIONS

4.1 Dynamic bending strength

Bending bars, 3.5x4.4x45mm, cut out from a hot isostatically pressed Al\(_2\)O\(_3\) -container (manufactured by ASEA, Sweden) were made available by NAGRA, Switzerland. The surface roughness is characterized by a mean value of peak to valley height of 0.27 \( \mu \)m and corresponding maximum value of 3.5 \( \mu \)m. The crack growth parameters of these specimens were to be determined and lifetime predictions made for some postulated tensile stresses. A salt solution (based on NAGRA AN/84-61) was specified as the test environment. The concentration in mg/l was

<table>
<thead>
<tr>
<th>Salt</th>
<th>Concentration</th>
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<tbody>
<tr>
<td>NaCl</td>
<td>8297</td>
</tr>
<tr>
<td>KCl</td>
<td>86</td>
</tr>
<tr>
<td>MgCl(_2) 6H(_2)O</td>
<td>22</td>
</tr>
<tr>
<td>SrCl(_2) 6H(_2)O</td>
<td>64</td>
</tr>
<tr>
<td>NaF</td>
<td>8</td>
</tr>
<tr>
<td>NaHCO(_3)</td>
<td>84</td>
</tr>
<tr>
<td>CaCl(_2) 2H(_2)O</td>
<td>3191</td>
</tr>
<tr>
<td>Na(_2)SO(_4)</td>
<td>2307</td>
</tr>
</tbody>
</table>

Prior to testing the Al\(_2\)O\(_3\) specimens were annealed for 2 hours at 1150\(^\circ\)C under vacuum conditions (10\(^{-5}\)bar). To determine the inert bending strength \( \sigma'_C \) a sample of 15 specimens was subjected to dynamic 4-point bending tests with a 20 mm inner span and a 40 mm outer span in air and a high stress rate of \( \sigma' = 350 \) MPa/s. The result is shown in Fig. 3 in a Weibull plot. For this purpose, the N strength values were arranged in ascending order and the cumulative distribution function \( F = i/(N+1) \) was allocated to the \( i \)-th value of \( \sigma'_C \). The results were plotted as \( \ln \ln 1/(1-F) \) vs \( \sigma'_C \) according to Eq. (10). The results of measurement were evaluated using the maximum likelihood method /7/ and they yielded the data

\[
m = 10.4 \quad \sigma'_0 = 369 \text{ MPa}
\]

The median value of the inert bending strength - i.e. the value corresponding to \( F = 0.5 \) - is \( \sigma'_C = 355 \) MPa.

4.2 Lifetime measurements

Lifetime measurements were performed in static 4-point bending tests
at 70°C in the salt solution specified in Section 4.1. The lifetimes obtained for different bending stresses are represented in Fig. 4. The bending stresses applied were 217, 173, 155 and 140 MPa. Lifetime tests exceeding a limit of 400 hours were discontinued and classified as "runthroughs". Only at the lowest load 5 runthroughs were observed. A Weibull analysis using the maximum likelihood method gives the following Weibull parameters m* and median values \( \hat{t}_f \) - i.e. the lifetimes at \( F = 0.5 \):

### Table 1

<table>
<thead>
<tr>
<th>( \sigma ) (MPa)</th>
<th>( m^* )</th>
<th>( \hat{t}_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>217</td>
<td>0.808</td>
<td>244 s = 0.068 h</td>
</tr>
<tr>
<td>173</td>
<td>1.316</td>
<td>38900 s = 10.8 h</td>
</tr>
<tr>
<td>155</td>
<td>0.861</td>
<td>1.41x10⁵ s = 39.06 h</td>
</tr>
<tr>
<td>140</td>
<td></td>
<td>1.58x10⁶ s = 440.2 h</td>
</tr>
</tbody>
</table>

The median lifetime \( \hat{t}_f \) for \( \sigma = 140 \) MPa was evaluated by extrapolation of the three lifetime data.

5. DETERMINATION OF CRACK GROWTH BEHAVIOUR

In Fig. 5 the median values of lifetime \( \hat{t}_f \) are plotted versus the bending stress applied. From the slope of the fitted straight line one obtains \( n \) by using Eq. (6) in a logarithmic formulation

\[
\ln \sigma = \frac{1}{n} \ln (B \sigma_c^{n-2}) - \frac{1}{n} \ln \hat{t}_f
\]

(slope = \(-1/n\), as

\[
n = 20
\]

For a lifetime of \( \hat{t}_f = 1 \) h one can conclude a corresponding applied stress \( \sigma' = 188.3 \) MPa. Eq. (18) yields

\[
\ln (B \sigma_c^{n-2}) = 104.76 \quad , \quad B=0.3914 \quad \text{MPa}^2\text{h}
\]

A second method of evaluating \( n \) is based on Eq (12a). By use of \( m = 10.4 \) and the \( m^*- \) data reported in Table 1 one obtains

### Table 2

<table>
<thead>
<tr>
<th>( \sigma' ) (MPa)</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>217</td>
<td>14.9</td>
</tr>
<tr>
<td>173</td>
<td>9.9</td>
</tr>
<tr>
<td>155</td>
<td>14.1</td>
</tr>
</tbody>
</table>
The mean value of $\bar{n} = 13$ confirms the relatively low $n$-value obtained by evaluation of Fig. 5.

In addition, the crack growth behaviour is analysed employing the method reported in Section 3b. Even if the distributions of the inert bending strength or the lifetime can only be described approximately by a Weibull distribution, the crack growth rates can still be derived from the individual strength and lifetime values. The procedure is also independent of the validity of Eqs. (5) and (6).

Figure 6 shows all measured lifetimes in dependence of the normalized stresses $\sigma/\sigma_c$. For each of the three test series the slope $d\ln(K_{II}/K_{IC})/d\ln t_f$ were determined. The investigated material showed straight lines. For each measured lifetime $t_f$ the related inert strength $\sigma_c$ corresponding to the same cumulative frequency is known. Then the $v-K_I$-data can be computed by application of Eq. (17) using $Y=2/\sqrt{n}$, $K_{IC} = 4$ MPa m$^{1/2}$. The $v-K_I$ dependency is depicted in Fig. 7. It can be expressed by a power law down to crack growth rates of $10^{-7}$ m/s. The fitted straight line is drawn in Fig. 7 and shows an exponent $n = 19$ and $B = 0.59$ MPa$^2$h in good agreement to the value obtained from Fig. 5.

6. COMPARISON WITH LITERATURE DATA

Since $n$, $B$, $m$ and $\sigma_c$ are known, it is possible to make lifetime predictions. The magnitude of $n$ is of very high importance for such predictions because $t_f \sim \sigma^{-n}$. For the investigated material $n$-values of the order of 200 and more were reported in /9/ in contrast to the data found in this investigation. The data of /9/ were obtained by Double-Torsion (DT) measurements carried out in so-called "Groundwater 1", which is representative of conditions in the Swedish granitic bedrock.

Deuerler, Knehans and Steinbrech /4/ have shown that ceramics with R-curve behaviour (i.e. $K_{IC}$ increases with crack extension) yield incorrect $n$-values if macroscopic cracks are taken into account. In the course of extension of large cracks (of the order of mm) the increasing $K_{IC}$ affects the $n$-values. In their experiments the authors found $n > 150$. After elimination of the influence of R-curve behaviour the crack growth exponent $n$ - describing pure subcritical crack growth - became $n \approx 41$. This was in good agreement with dynamic bending tests where $n \approx 50$ was found.

7. LIFETIME PREDICTIONS

Lifetime predictions can be made by use of Eq. (6) and Eq. (10). From Eq. (10) it follows

$$\sigma_c = \sigma'_c \exp \left[ \frac{1}{m} \ln \ln \frac{1}{1-F} \right]$$

(19)

and by insertion into Eq. (6), one obtains
\[ t_f = B \sigma_o^{n-2} \exp\left( \frac{m^2}{n} \ln \ln \frac{1}{(1-F)} \right) \sigma^{-n} \] (20)

For ultimate storage container an admissable failure probability of \( F = 10^{-3} \) is assumed. Two questions are of interest:

a) A minimum lifetime of 1000 years is required. What are the allowa-
ble tensile stresses?

\[ t_f = 8.76 \cdot 10^5 \text{h} \quad \text{and} \quad \sigma_o = 369 \text{ MPa} \]

\[ \sigma_{\text{max}} = 48.2 \text{ MPa} \]

b) In the second question residual stresses \( \sigma_{\text{res}} \) caused by manufacture
are supposed. The expected lifetime is:

1. \( t_f = 4.4 \cdot 10^{10} \text{ years} \quad \text{for} \quad \sigma_{\text{res}} = 20 \text{ MPa} \)
2. \( t_f = 6.1 \cdot 10^5 \text{ years} \quad \text{for} \quad \sigma_{\text{res}} = 35 \text{ MPa} \)
3. \( t_f = 484 \text{ years} \quad \text{for} \quad \sigma_{\text{res}} = 50 \text{ MPa} \)

Relation (20) is represented in Fig. 8 and the examples are marked
in it.

The lifetime predictions mentioned in this chapter are performed
for specimen of constant sizes. It is known that strength and life-
time will decrease if volumina and surface of predicted construc-
tion are larger than those of measured bending specimens. Conse-
quently, the allowable stresses and attainable lifetimes become
lower. For exact calculations the stress distribution in the con-
tainer wall has to be known.

8. INFLUENCE OF SURFACE ROUGHNESS ON BENDING STRENGTH

As mentioned in Section 1, the surface roughness was characterized by
a maximum peak-to-valley height of 3.5\( \mu \text{m} \) and a corresponding mean
value of 0.27\( \mu \text{m} \). It was of interest to know the influence of the
surface quality on the inert bending strength. Therefore, three sur-
faces of a series of bending bars were polished which produced a maxi-
mum peak-to-valley height of 0.046\( \mu \text{m} \) and a mean roughness of only
0.004\( \mu \text{m} \). After annealing under vacuum conditions the inert bending
strength was measured again. As shown in Fig. 9, an increment of ap-
proximatively 40% in bending strength can be observed, but the Weibull
modulus \( m \) is hardly influenced. This suggests that the relevant crack
size had been reduced by a factor of approximately 2 in the course of
the polishing to 0.27\( \mu \text{m} \).

9. SUMMARY

A series of bending bars cut out from a hot isostatically pressed
alumina container were investigated to determine the crack growth data
in a salt solution at 70°C. The crack growth behaviour was studied in
4-point static bending tests. The results are:
- The Weibull parameters of the inert bending strength distribution are \( m = 10.4; \sigma'_o = 369 \text{ MPa}, \sigma'_c = 355 \text{ MPa}; \)

- Lifetime measurements yield a low crack growth exponent of \( n = 20 \) obtained by evaluation of the relation \( \hat{t}_f = f(\sigma); \)

- A new method was applied to determine the \( v-K_1 \)-relationship from lifetime measurements.

- The \( v-K_1 \)-curve can be expressed by a power law down to crack growth rates of \( 10^{-11} \text{ m/s} \) with an exponent of \( n = 19. \)

- An analysis of Weibull moduli \( m_t \) confirms those surprisingly low \( n \)-values.

- The influence of R-curve effects on \( n \)-values measured in DT-tests is discussed on the basis of results published in the literature.

- A formula is given to allow lifetime predictions for different failure probabilities.

- Finally, the influence of surface quality on bending strength is mentioned to give an impression of attainable strength data.
10. REFERENCES


/9/ W. Hermansson: "Determination of slow crack growth in isostatically pressed Al2O3" in: Ref. /1/
11. FIGURES

Fig. 1: Schematic of lifetimes ranked in increasing order as a function of initial load.

Fig. 2: $v$-$K_I$-curve evaluated from lifetime measurements shown in Fig. 1.

Fig. 3: Inert bending strength $\sigma_c$ measured in dynamic 4-point bending tests in air with $\dot{\sigma} = 350$ MPa/s.

Fig. 4: Lifetimes $t_f$ measured in static 4-point bending tests in salt solution.

Fig. 5: Median values $\hat{t}_f$ of lifetimes from Fig. 4 in dependence of applied bending stresses $\sigma$.

Fig. 6: Individual lifetimes $t_f$ from Fig. 4 in dependence of applied bending stresses $\sigma$ normalized on individual inert strength $\sigma_c$.

Fig. 7: $v$-$K_I$-relationship obtained from data of Fig. 6 by application of Eq. (17).

Fig. 8: Nomograph for lifetime predictions.

Fig. 9: Influence of surface quality on bending strength
- maximum peak-to-valley height 3.5 $\mu$m (mean value 0.27 $\mu$m)
- maximum peak-to-valley height 0.046 $\mu$m (mean value 0.004 $\mu$m).
Fig. 1: Schematic of lifetimes ranked in increasing order as a function of initial load.

Fig. 2: $v$-$K_I$-curve evaluated from lifetime measurements shown in Fig. 1.
Fig. 3: Inert bending strength $\sigma'_c$ measured in dynamic 4-point bending tests in air with $\dot{\sigma}' = 350$ MPa/s.
Fig. 4: Lifetimes $t_f$ measured in static 4-point bending tests in salt solution.
Fig. 5: Median values $\hat{t}_f$ of lifetimes from Fig. 4 in dependence of applied bending stresses $\sigma$. 
Fig. 6: Individual lifetimes $t_f$ from Fig. 4 in dependence of applied bending stresses $\sigma$ normalized on individual inert strength $\sigma_c$. 

$\frac{\sigma}{\sigma_c} = \frac{K_f}{K_x}$
Fig. 7: \( v-K_1 \)-relationship obtained from data of Fig. 6 by application of Eq. (17).
Fig. 8: Nomograph for lifetime predictions.
**Fig. 9:** Influence of surface quality on bending strength

- Maximum peak-to-valley height 3.5 μm (mean value 0.27 μm)
- Maximum peak-to-valley height 0.046 μm (mean value 0.004 μm).